

Title of the Talk: Subgroup counts and structure of finite groups.

Abstract of the Talk: The number of subgroups and the number of cyclic subgroups are

natural combinatorial invariants of a finite group. We investigate how restrictions on these quantities, together with the number of distinct prime divisors of $|G|$, enforce nilpotency,

supersolvability, and solvability of G . These criteria improve earlier results that relied solely on the total number of subgroups, and they are sharp in the sense that for each bound there exist non-nilpotent (resp. non-supersolvable, non-solvable) groups attaining the bound.

